Centric Selection: a Way to Tune the Exploration/Exploitation Trade-off

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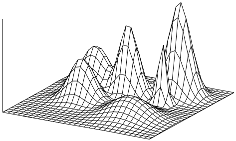
Exploration / exploitation tradeoff

One of the fondamental problem in EA



- Too much exploitation : population get stuck in local optima
- Too much exploration : random walk on fitness landscape

$\longrightarrow$ exploration / exploitation tradeoff

Selective Pressure in EA

Definition

the ability of best solutions to conquer the whole population

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$$\begin{aligned} & \text{Selective Pressure} \\ \Longrightarrow \quad & \text{Population Diversity} \\ \Longrightarrow \quad & \text{Exploration / Exploitation tradeoff} \end{aligned}$$

Selective Pressure in EA

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the ability of best solutions to conquer the whole population

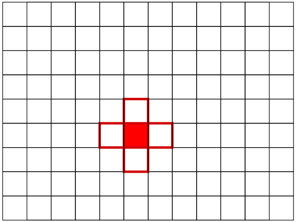
$$\begin{aligned} & \text{Selective Pressure} \\ \Longrightarrow \quad & \text{Population Diversity} \\ \Longrightarrow \quad & \text{Exploration / Exploitation tradeoff} \end{aligned}$$

Some methods which try to tune selective pressure :

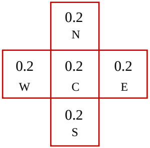
- Island models
- Sharing methods
- Cellular Genetic Algorithm
- ...

Cellular Genetic Algorithms
spatial structured population

One solution in each cell



- Neighborhood : Von Neumann, ...



Cellular Genetic Algorithms

spatial structured population

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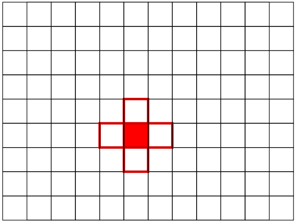

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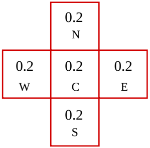

- Genetic operators are local:
  Selection of parents within the neighborhood (tournament selection,...)
- After selection, crossover, mutation:
  Replacement of the solution in C if better

Cellular Genetic Algorithms

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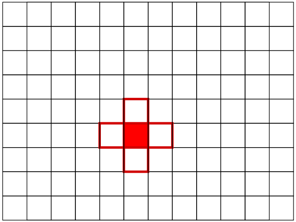

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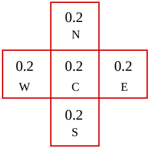

- Genetic operators are local:
  Selection of parents within the neighborhood (tournament selection,...)
- After selection, crossover, mutation:
  Replacement of the solution in C if better
- Overlapping neighborhoods : implicit mechanism for migration → control selective pressure

Goal of this work

Goal is to establish a relation between:

- Selective pressure on the population
- the effects of recombination and mutation operators

$\Rightarrow$ in order to explain and find an optimal exploration/exploitation trade-off

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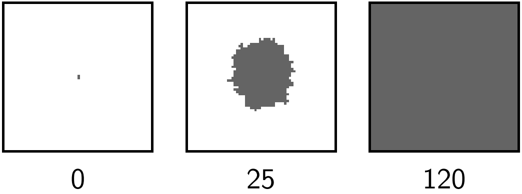
We propose:

- New selection scheme able to control the selective pressure: Centric Selection
- Theoretical model which takes into account the effects of stochastic variations: Punctuated Equilibria Model

Mesure of selective pressure

The Takeover Time [Goldberg 90] is the time it takes for the single best solution to conquer the whole population when the only active operator is selection.



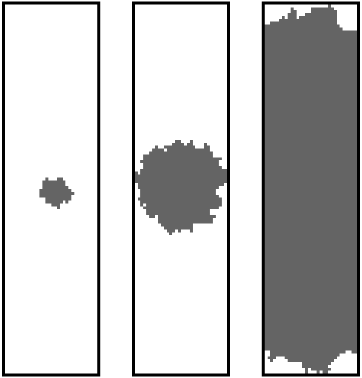
- Long takeover time : low selective pressure
- Short takeover time : high selective pressure

Grid Shape and takeover time

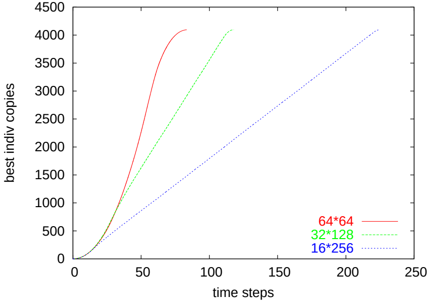
Pop. size $= 2^{12}$	Avg Takeover Time
64×64	83.4
32×128	117.8
16×256	225.0
8×512	449.7
4×1024	937.1
2×2048	2101.2

- Square grid : takeover time is short
  High selective pressure
- Narrow grid : takeover time is long
  Low selective pressure

Spreading of best solution: the growth curve



Spreading of the best solution

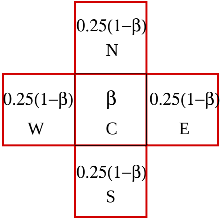


- 3 times in spreading[Giacobini 05]: quadratic, linear, quadratic (it is exponential for panmitic EA)

Centric Selection

Principe

Modify the probability to participate to the tournament



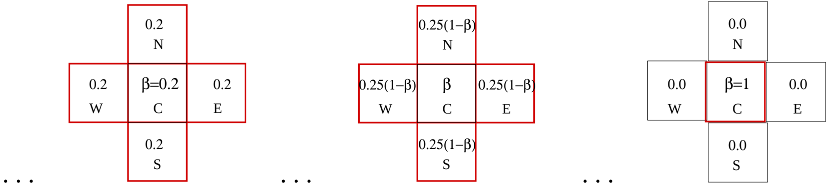
Probability to participate to the tournament :

- cell center : $p_c = \beta$
- north, south,

east or west cell : $p_s = p_n = p_e = p_w = \frac{1}{4}(1 - \beta)$

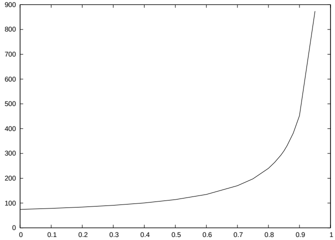
β tunes the centric selection:
it is possible to slow down and control the selection pressure in a continous isotropic manner

Centric Selection : isotropic "Fuzzy" neighborhood



$\beta = 0.2$	β	$\beta = 1$
Von Neumann Neighborhood	"fuzzy" isotropic Neighborhood	parallel Hill-Climbing

Centric Selection and Selective Pressure
Takeover time



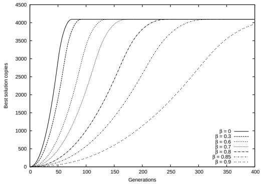
Average takeover time as a function of β for a 64×64 grid

- Takeover time is not defined for $\beta = 1$ (no communication between cells),
- Selective pressure drops when the value of β increases.

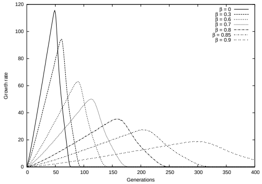
Centric Selection and Selective Pressure

Growth curves and the growth rates

Growth curves on 64 × 64 grid



Corresponding growth rate



Two stages:

- First: linear growth rate,
- Second: quadratic growth rate.

Interpretation:

- First: isotropic diffusion, roughly propagates describing an obtuse square,
- Second: the sides are reached, the dynamic changes.

Quadratic Assignment Problem (QAP)

Problem of assigning a set of N facilities to a set of N locations with given distances between the locations d_{ij} and given flows between the facilities f_{ij}

$$\Phi(p) = \sum_{i=1}^{N} \sum_{j=1}^{N} d_{p(i)p(j)} f_{ij}$$

where $p(i)$ is the location of facility i

$\Longrightarrow$ Find the permuation p which minimize the total flow Φ

NK fitness landscapes

$$f(x) = \frac{1}{N} \sum_{i=1}^{N} f_i(x_i, x_{i_1}, \ldots, x_{i_K})$$

- N : length of the bit string
- $K \leq N - 1$ number of interactions
- $x_i \in \{0, 1\}$
- $\{i_1, \ldots, i_K\} \subset \{1, \ldots, i-1, i+1, \ldots, N\}$
- $f_i : \{0,1\}^{K+1} \rightarrow [0,1]$ choosen at random

Results on QAP

Avg. results and std.dev. on QAP instances

Instance	Std cGA	Best avg. results	Optimal β
Nug30	$6178_{[28]}$	$6144_{[14]}$	0.88
Tai40a	$3.23 \times 10^6_{[14343]}$	$3.21 \times 10^6_{[12000]}$	0.84
Sko42	$15969_{[75]}$	$15909_{[34]}$	0.82
Tai50a	$5.092 \times 10^6_{[20721]}$	$5.080 \times 10^6_{[13372]}$	0.82
Tai60a	$7429118_{[27760]}$	$7385390_{[19391]}$	0.86

$\Rightarrow$ The optimal value of β is arround 0.86

Results on NK landscapes

Avg. performances and std.dev. on NK instances with $N = 32$

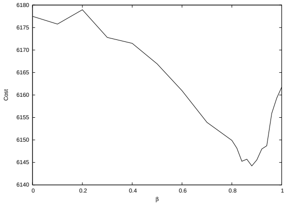
K	Std cGA	Best avg. results	Optimal β
2	$0.734329_{[0]}$	$0.734329_{[0]}$	$[0, 1]$
4	$0.79597_{[0.003]}$	$0.798197_{[0]}$	1
6	$0.782934_{[0.01]}$	$0.799124_{[0.003]}$	1
8	$0.771277_{[0.01]}$	$0.789103_{[0.004]}$	1
10	$0.763510_{[0.01]}$	$0.785115_{[0.003]}$	1
12	$0.750043_{[0.01]}$	$0.774479_{[0.009]}$	1

$\Rightarrow$ The optimal value of β is 1.0

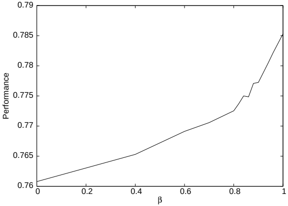
Optimal explotation/exploiration tradeoff

- The optimal exploration/exploration tradeoff is different according to the class of problem:

Avg. Performances according to β:



nug30



NK with $N = 32$ and $K = 10$

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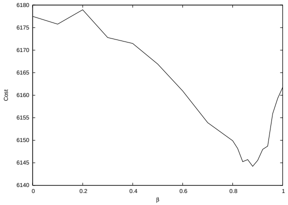

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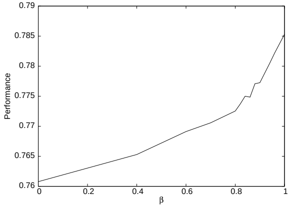


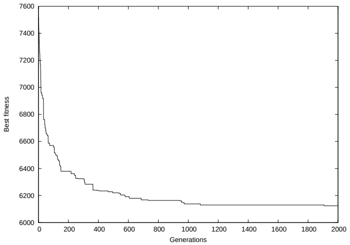
NK with $N = 32$ and $K = 10$

Questions:

- How to explain theoreticaly the difference ?
- How to find an optimal trade-off ?

From Equilibrium model to Punctuated Equilibria Model

Typical run on a minimization problem:



Punctuated equilibria dynamic:

- Long period without improvement
- Rapid change: a new best solution is found

From Equilibrium model to Punctuated Equilibria Model

	Equilibrium Model:	Punctuated Equilibria Model:
Goal	To study selective pressure	To study selective pressure and the effect of variation operators
Simul. Ope.	Only the selection operator is active	+ Probability to find a new best solution by crossover and mutation
Init. pop.	Only one best solution in the population	Only one best solution in the population
Observ.	Takeover time, growth curve	Probability and time to find a new best solution

Punctuated Equilibria Model

- Initialization: cEA initialized with random solutions, the best solution is unique.
- Selection operator: centric selection
- Simulation of crossover and mutation operator: probabilities to find a new best solution according to the mating type
- Three different types of matings:
    - between two copies of the best solution (mating 11),
    - between one copy of the best solution and one sub-optimal solution (mating 01)
    - between two sub-optimal solutions (mating 00).
- Probabilities P_{11}, P_{01} and P_{00} that matings of type 11, 01 and 00 produce a new best solution

Punctuated Equilibria Model

With this model,

Probability of finding a new best solution at a given gen. t

$$p(t) = 1 - (1 - P_{00})^{n_{00}(t)}(1 - P_{01})^{n_{01}(t)}(1 - P_{11})^{n_{11}(t)}$$

where $n_{00}(t)$, $n_{01}(t)$ and $n_{11}(t)$ are the number of matings of each type for the generation t.

Average time to find a new best solution

$$E = \sum_{t \geq 1} tp(t)$$

Punctuated Equilibria Model

With this model,

Probability of improving the best solution in T generations

$$P = 1 - (1 - P_{00})^{\Sigma_{00}(T)}(1 - P_{01})^{\Sigma_{01}(T)}(1 - P_{11})^{\Sigma_{11}(T)}$$

with $\Sigma_{ij}(T) = \sum_{t=1}^{T} n_{ij}(t)$

Intuitively, ideal selection process maximizes the Σ_{ij} which have the higher P_{ij}

Punctuated Equilibria Model: ideal trade-off

$$P = 1 - (1 - P_{00})^{\Sigma_{00}} (1 - P_{01})^{\Sigma_{01}} (1 - P_{11})^{\Sigma_{11}}$$

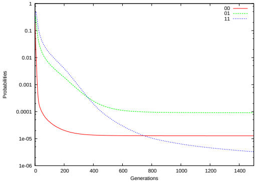
Optimal value β^* of control parameter β

$$\frac{dP}{d\beta}(\beta^*) = 0$$

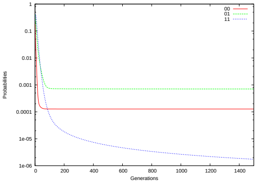
- PE Model explains the tradeoff between:
  - Exploitation: selection pressure given by Σ_{ij} (control by β)
  - Exploration: effect of variation operator given by P_{ij} (problem dependent)
- If it is possible to have a model of $\Sigma_{ij}(\beta)$, it would be possible to calculate the optimal β^* as a function of P_{ij}.

Estimated P_{ij} on QAP and NK landscapes

P_{ij} for the QAP problem nug30



NK with $N = 32$ and $K = 10$



Method:

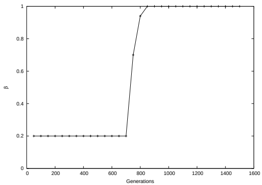
- Estimation of P_{ij} with a Bayesian process during the runs.
- Average the values obtained by generations over 500 runs.

Results:

- For both: $P_{01} > P_{00}$ and P_{11} curve intercepts the others
- The intercept point is not the same according to the class of the problems

Theoretical optimal value of β

QAP problem nug30



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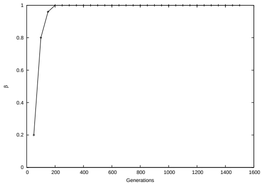
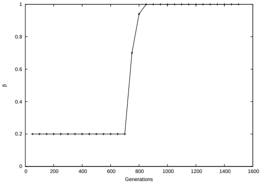


- QAP:
  - Transition between the generation 700 and 850
  - Before optimal value is $\beta = 0.2$
  - After optimal value is $\beta = 1.0$
- NK:
  - Optimal value increases very fast
  - After a short transition: optimal value is 1.0

Theoretical optimal value of β

QAP problem nug30



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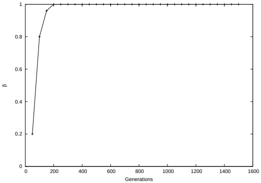

According to the model,
when β is constant,
the optimal value β^* should be:

- QAP: intermediate and higher than 0.7
- NK: very high around 1.0

$\Rightarrow$ Which correspond to the experimental observation

Conclusion and Future Works

We have proposed:

- New model of selection in cellular GA: centric selection
- Control the selective pressure with a continous parameter
- New theoretical model to explain exploitation/exploration trade-off: Punctuated Equilibria Model

Future works :

- Apply the PE model to other types of EA
- Increases the accuracy of PE model to take into account other types of matings
- Auto-adaptation: predict the optimal value of β according to an online estimation of P_{ij}