



The Road to VEGAS: Guiding the Search over Neutral Networks

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M-E. Marmion, C. Dhaenens, L. Jourdan, A. Liefoghe, S. Verel

Firstname.lastname@inria.fr

INRIA Lille-Nord Europe

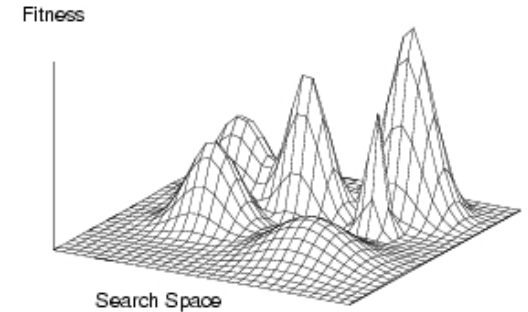
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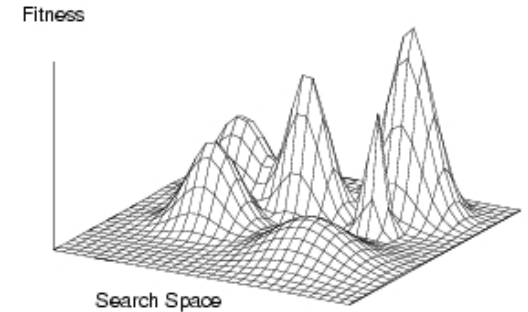
Motivations

- Fitness **landscapes**
 - To analyze the structure of the search space
 - Understanding the problem structure to design efficient search methods
- Many combinatorial optimization problems involve **neutrality** (robot controller, planning problem, learning problem, protein folding...)
- Can we and how to exploit this neutrality?



Motivations

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Objectives

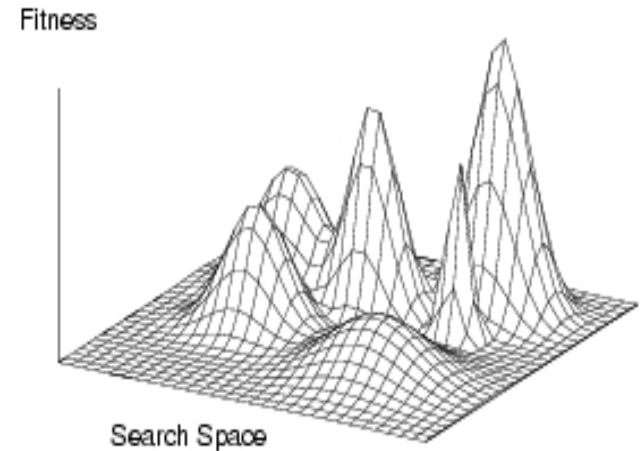
- How to **exploit** neutrality?
- How to **guide the search** adaptively over a neutral network?
- What is **Exploration / Exploitation** tradeoff of neutral networks?

→ **VEGAS** : Varying **E**volvability **G**uided **A**daptive **S**earch

Fitness Landscape

FiL (S, N, f)

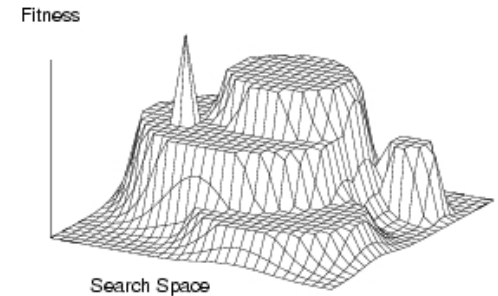
- S Search space
all feasible solutions
- $N : S \rightarrow 2^S$ Neighborhood relation
N(s) : neighborhood, $s' \in N(s)$: neighbor of s
- $f : S \rightarrow R$ Evaluation function
Fitness values assigned to solution



local optima, ruggedness, etc.

Neutrality properties

- **Neutral Network (NN) - Plateau**
 - Connected sub-graph
 - Vertices: equivalent solutions (same fitness value)
 - Edges: Neighborhood structure
- **Degree of neutrality**
 - Number of neighbors with the same fitness value
$$\mathcal{N}_n(s) = \{s' \in \mathcal{N}(s) \mid f(s') = f(s)\}$$
- **Portal (exit solution)**
 - Solution from NN with at least one improving neighbor



Guiding the search on NN

- How to **guide** the search?
 - Consider all solutions with the same fitness
 - Estimate the evolvability of solutions
 - Select the most promising solution



Estimate Evolvability

Altenberg : the ability of random variations to sometimes produce improvement

Existing measures of evolvability:

- Average, max fitness values from the neighborhood
- Probability to increase
- Neutral degree...

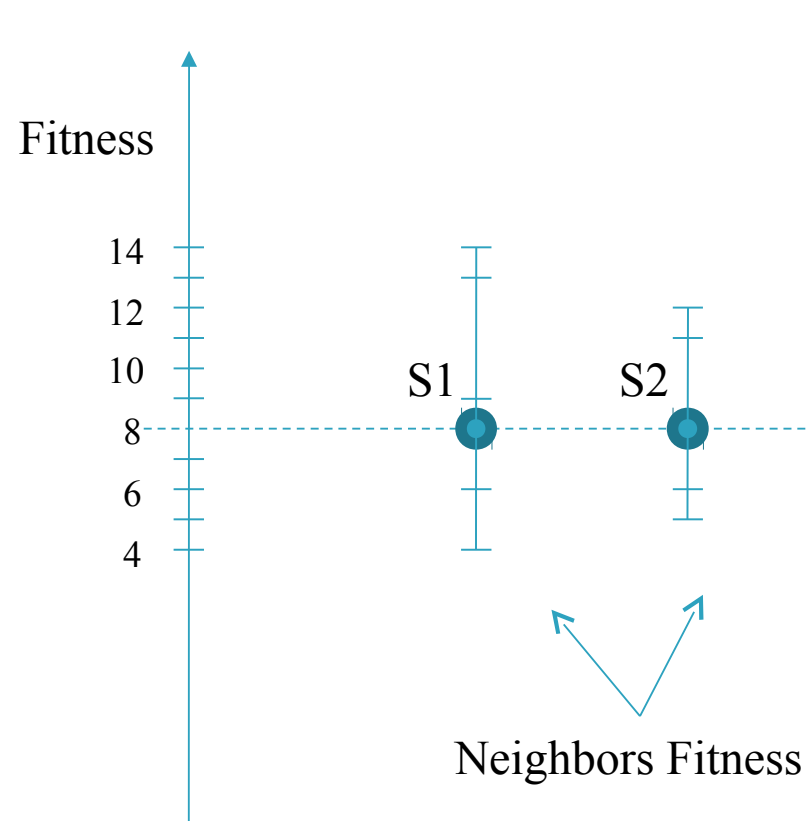
Our approach

- Inspired by the « Area Under Curve » (AUC) scheme used for operator selection
- Neighborhood sampling

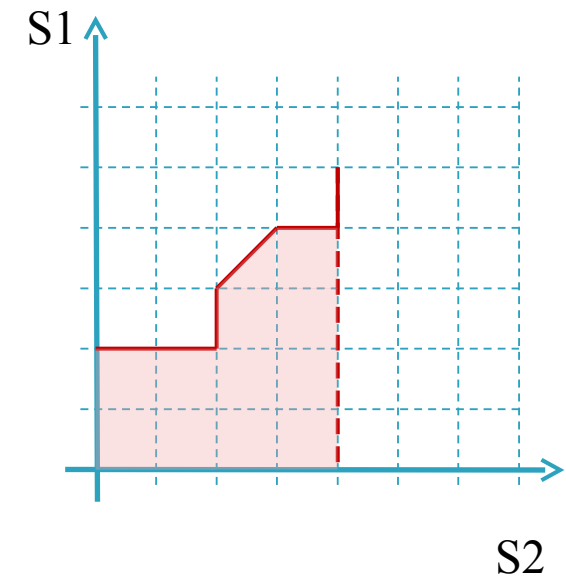
Altenberg, L.: The evolution of evolvability in genetic programming. In Kinnear, Jr., K.E., ed. : Advances in Genetic Programming. MIT Press (1994) 47–74

Álvaro Fialho, Marc Schoenauer and Michele Sebag. Toward Comparison-based Adaptive Operator Selection. In J. Branke et al., eds.: "GECCO'10: Proc. 12th Annual Conference on Genetic and Evolutionary Computation", ACM Press : p. 767-774. July 2010

Estimate Evolvability



Fitness	Solution
14	S1
13	S1
12	S2
11	S2
9	S1
6	S1
6	S2
5	S2
4	S1



$$\text{AUC}(S1)=11.5$$

Select the most promising solution

- ▶ Exploration vs. exploitation
 - Multi-armed bandit
 - Upper Confidence Bound strategy (UCB)

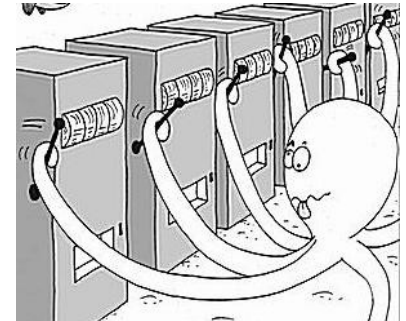
$$\arg \max_{i=1..K} \left(\hat{r}_{i,t} + C \sqrt{\frac{\log \sum_k n_{k,t}}{n_{i,t}}} \right)$$

K : number of arms

\hat{r}_i : credit of arm i

n_i : number of applications of arm i

C : controls the trade-off (exploitation vs. exploration)



Álvaro Fialho, Marc Schoenauer and Michele Sebag. *Toward Comparison-based Adaptive Operator Selection*. In J. Branke et al., eds.: "GECCO'10: Proc. 12th Annual Conference on Genetic and Evolutionary Computation", ACM Press : p. 767-774. July 2010

Select the most promising solution

$$\arg \max_{i=1..K} \left(\hat{r}_{i,t} + C \sqrt{\frac{\log \sum_k n_{k,t}}{n_{i,t}}} \right)$$

At iteration t :

→ Arms = sampling from NN

K : sample size

→ Credit assignement based on evolvability

\hat{r}_i : AUC of solution i

→ Number of sampled neighbors

n_i : evaluated neighbor

→ Exploitation / Exploration trade-off parameter

C : small (exploitation), large (exploration)

Management of the NN sample:

- Add equivalent solutions
- Delete solutions when $n_j = |N(s)|$

VEGAS

$S = \{s_0\}$

WHILE $\exists s \in S$ such that s is not visited do

| $s \leftarrow \text{select}(S)$

| Choose a solution $s' \in N(s)$ at random (no repetition)

| IF $f(s) < f(s')$ THEN

| | $S \leftarrow \{s'\}$

| ELSE IF $f(s) = f(s')$ THEN

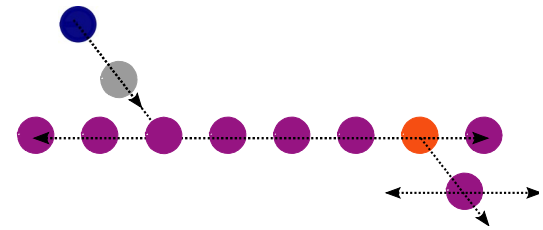
| | $S \leftarrow S \cup \{s'\}$

| END IF

| Update $\text{rewards}(s, s')$

END WHILE

Return $s \in S$



NKq fitness landscapes

$$f(x) = \frac{1}{N} \sum_{i=1}^N f_i(x_i, x_{i_1}, \dots, x_{i_k})$$

- N : length of the bit string, $x_i \in \{0, 1\}$
- $K \leq N-1$ number of interactions (epistasis, non-linearity)
- q number of possible values (neutral degree level)
- $\{i_1, \dots, i_k\} \subset \{1, \dots, i-1, i+1, \dots, N\}$
- $f_i: \{0, 1\}^{K+1} \rightarrow [0, q) \cap \mathbb{N}$ chosen at random

$x_1 x_2 x_4$	f_1
000	2
001	1
010	0
011	0
...	...

$x_1 x_2 x_3$	f_2
000	1
001	2
010	2
011	0
...	...

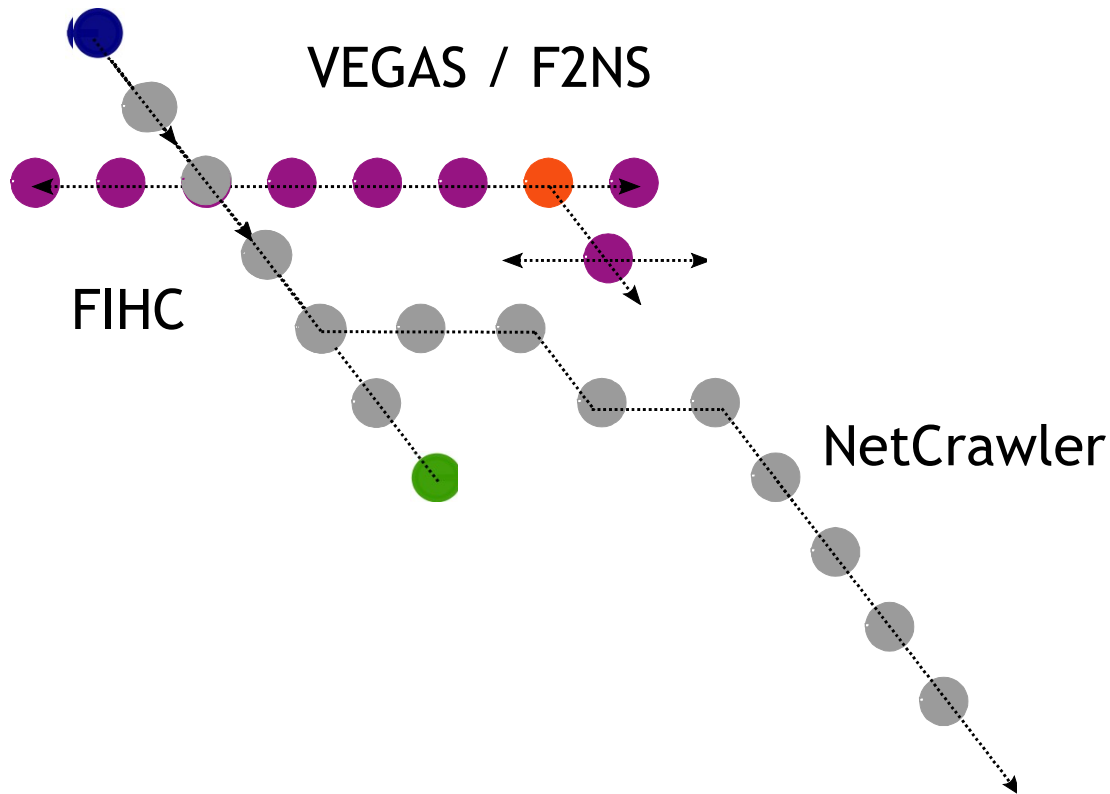
$x_2 x_3 x_4$	f_3
000	2
...	...
101	0
110	1
111	1

$x_1 x_2 x_4$	f_4
000	0
001	2
010	1
011	0
...	...

Experimental design

- Test problems
 - NKq N=64, $K \in \{2,4,6,8\}$, $q \in \{2,3,4\}$
- Neighborhood
 - 1 bit-flip
- Comparisons with
 - FIHC: First Improvement Hill-Climbing
 - NC: NetCrawler (HC which accepts if $f(s) \leq f(s')$)
 - F2NS: Fair Neutral Network Search (Select = random)
- Parameters
 - Stopping criteria: max number of eval. 10^5
 - 100 experiments/algorithm
- VEGAS
 - $C \in \{10^{-4}, 10^{-3} \dots 10^1, 10^2, 5 \cdot 10^2\}$

Dynamics of compared methods



FIHC

Without neutrality

NetCrawler

With Neutrality

NN sample size = 1

F2NS

With Neutrality

NN sample size > 1

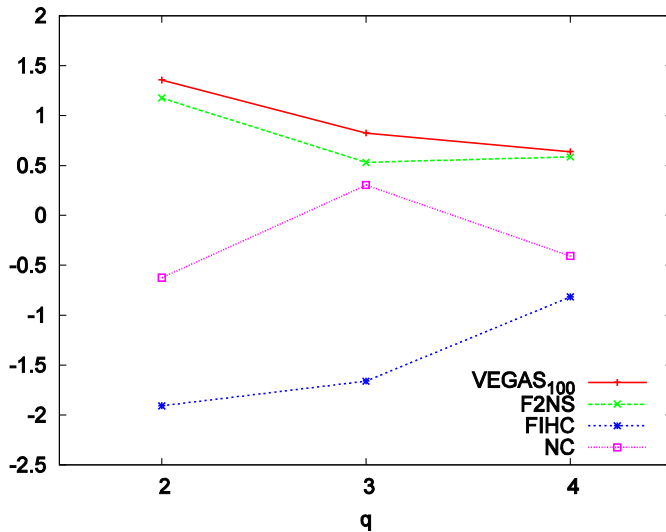
VEGAS

With Neutrality

NN sample size > 1

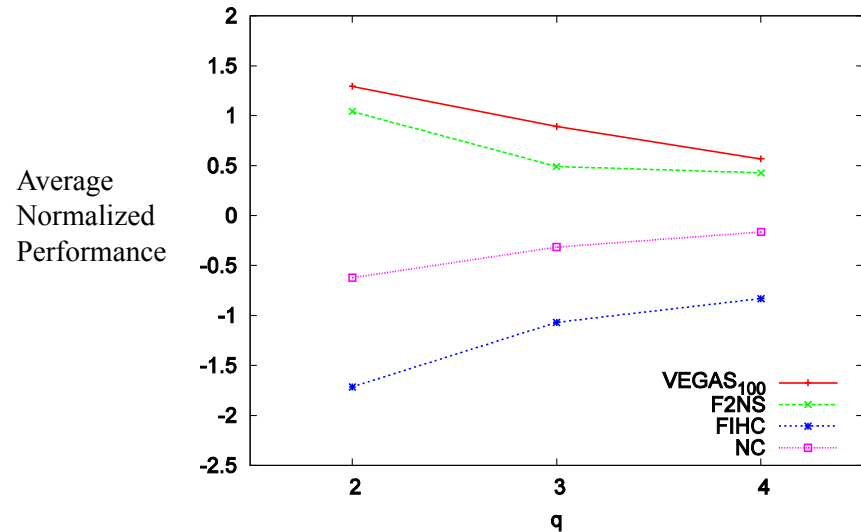
With Evolvability

Performance



$K = 4$

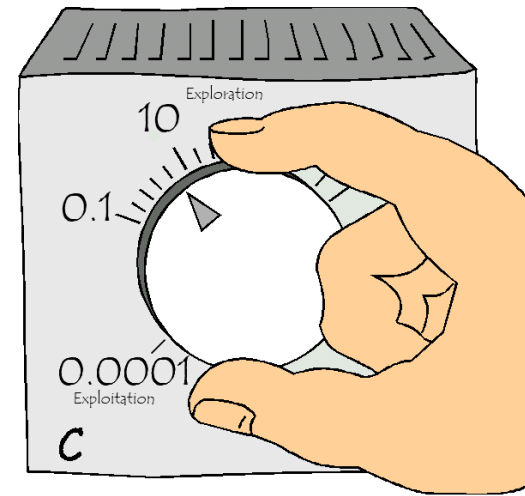
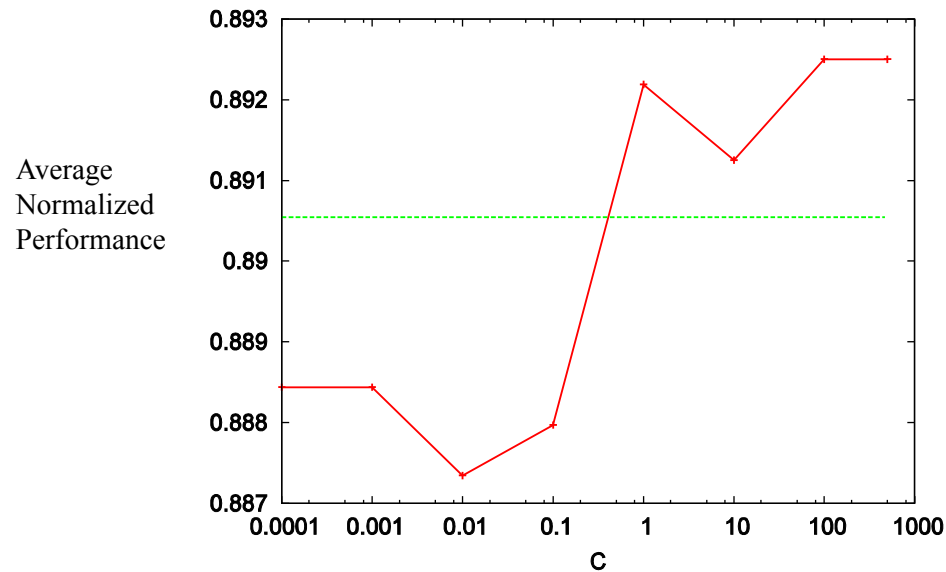
Neutrality ?
NN sample size > 1 ?
Evolvability ?



$K = 8$

FIHC \ll NC
NC \ll F2NS, VEGAS
F2NS $<$ VEGAS₁₀₀

Impact of parameter C



C controls the **Trade-off** Exploration vs. Exploitation?

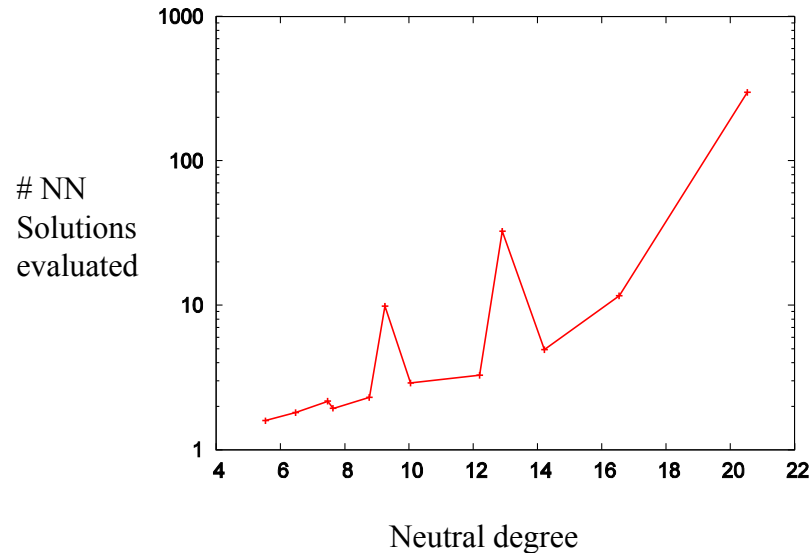
small C → Importance of evolvability

= **Exploitation**

high C → Towards less visited solutions

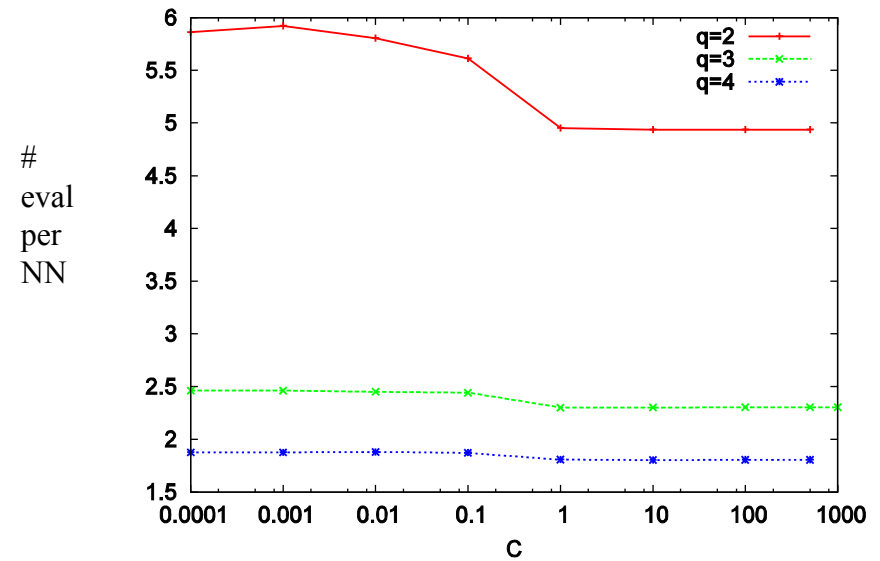
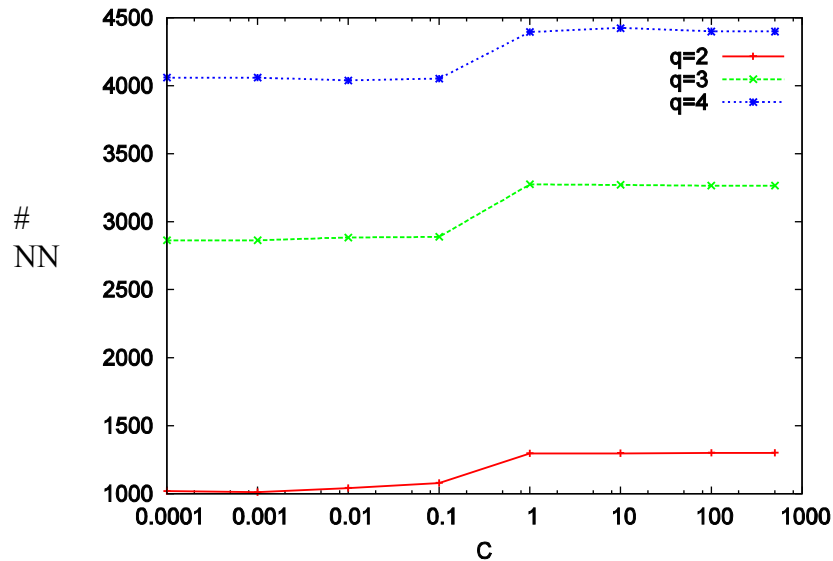
= **Exploration**

Impact of neutrality



NN Solutions evaluated on NN increases exponentially with neutral degree

Exploration vs Exploitation



$C > 1$ (exploration) \longrightarrow

more NN are sampled,
few evaluations on NN

$C < 1$ (exploitation) \longrightarrow

few NN are sampled,
more evaluations on NN

Conclusion

- Algorithm to exploit **neutrality**
- Adaptive **balance** exploration / exploitation of neutrality
- **Evolvability**-guided search

→ VEGAS

- Multi-Armed Bandit
 - Evolvability
 - A single parameter to control the exploration and exploitation trade-off of NN
-
- **Open issues**
 - Other evolvability measures?
 - Flowshop scheduling?



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