On the Effect of Connectedness for Biobjective Multiple and Long Path Problems

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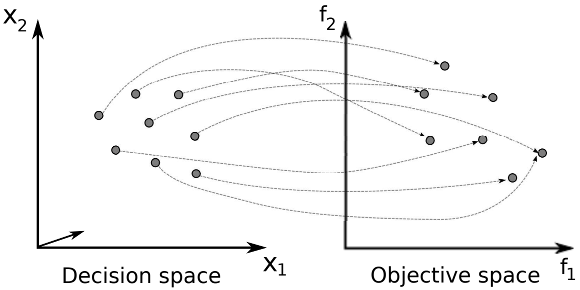
Multiobjective combinatorial optimization

Multiobjective combinatorial optimization (MoCO) problem

- $\mathcal{X}$: enumerable set of feasible solutions in the *decision space*
- $M \geq 2$ objective functions $(f_1, f_2, \ldots, f_M)$
- $\mathcal{Z} = f(\mathcal{X}) \subseteq \mathbb{R}^M$: set of feasible outcome vectors in the *objective space*

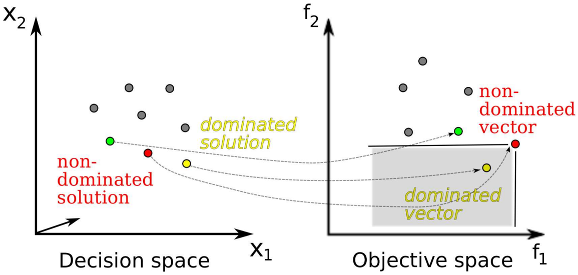


Multiobjective combinatorial optimization

Pareto dominance between solutions (maximization)

A solution $x \in \mathcal{X}$ dominates a solution $x' \in \mathcal{X}$ iff

- $\forall i \in \{1, 2, \ldots, M\}$, $f_i(x') \leq f_i(x)$
- $\exists j \in \{1, 2, \ldots, M\}$ such that $f_j(x') < f_j(x)$



Efficient set (or non-dominated set or Pareto optimal set)

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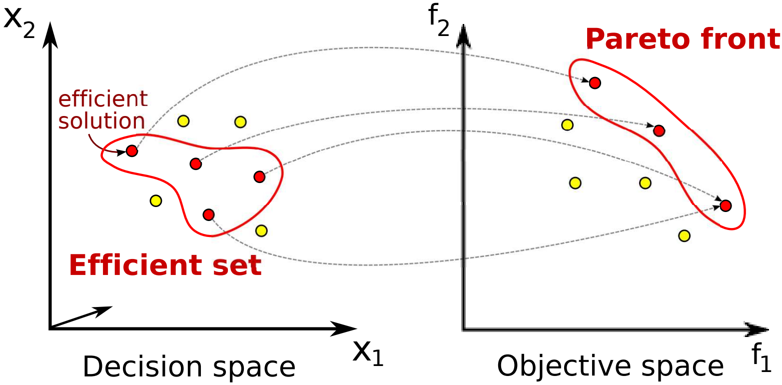
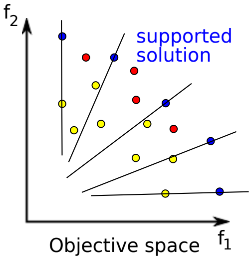
Set of non-dominated solutions

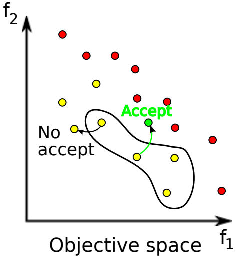


How to solve MoCO problems with a local search algorithm ?

Two main classes :

- Scalar approaches : multiple scalarized aggregations of the objective functions, only able to find a subset of efficient solutions (supported solutions)
- Pareto-based approaches : directly or indirectly focus the search on the Pareto dominance relation

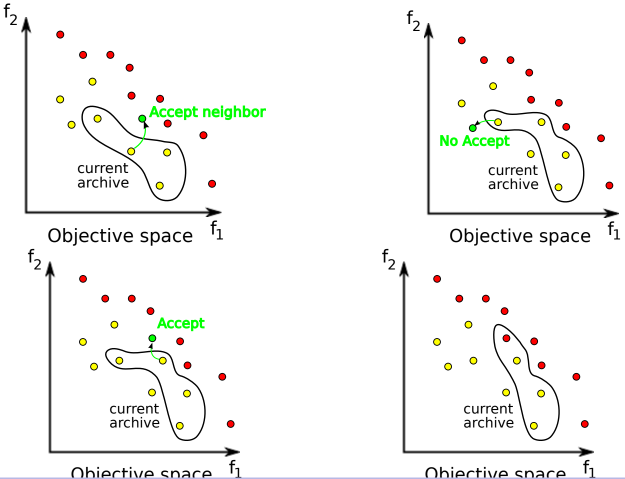




Example of Pareto-based approach

Pareto Local Search [Paquete et al., 2004]

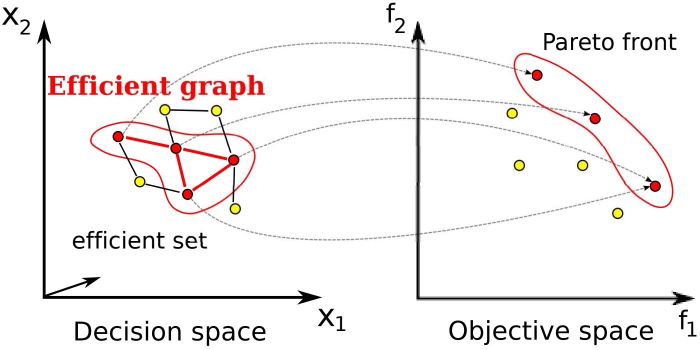
- Archive of mutually non-dominated solutions
- Iteratively improve this archive by exploring the neighborhood



Connectedness

Efficient graph [Ehrgott and Klamroth, 1997]

- $G = (V, E)$ where V consists of all efficient solutions and E connects two solutions if they are neighbors
- Efficient set is connected if G is connected



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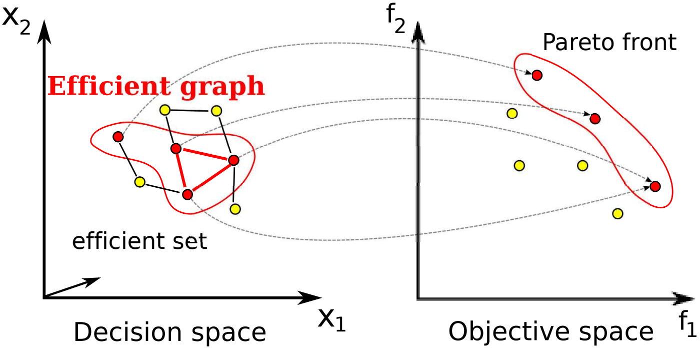
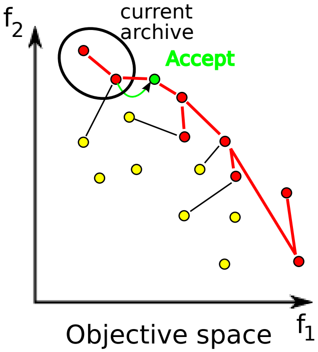


Two-phase approaches when connectedness is hold

- From an efficient solution (find with a scalar approach for example)
- Iteratively find a new efficient solution by exploring the neighborhood (Pareto-based approach)



Claims

- Connectedness : strong motivation on the LS design
- Structure of this efficient set : crucial role for LS design [Gorski et al., 2006]

Main difficulties in MoCO

Why identifying the efficient set is **difficult** [Ehrgott, 2005] ?

- NP-complete : deciding if a solution is efficient is NP-complete for most MoCO problems (even if no single-objective counterpart is NP-hard)
- Intractability : number of efficient solutions (non-dominated vectors) typically exponential in the problem size

⇒ Identify a "good" efficient set **approximation**, such that :

(*i*) close to Pareto front

(*ii*) well-spread along the Pareto front

Intractability : could be catastrophic for two-phase approaches, or Pareto-based approaches when the goal is not to enumerate the whole efficient set...

Motivations of this work

The connectedness is not the holy grail

We would like to show that :

When the MoCO problem is intractable :

- Connectedness is not relevant to explain the difficulty, and to improve the design of LS

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Even more...

- Reconsiderer the relevant characteristics of MoCO problems which can explain the dynamics of algorithms, difficulty of instances, help to design more efficient methods

Long k-Path problems : an old story in mono-objective

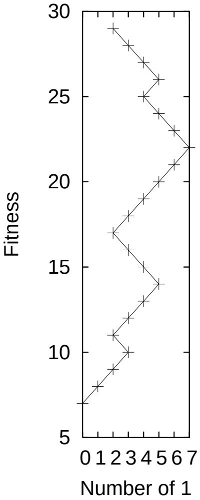
Goal [Horn et al., 1994]

Unimodal problem is not always easy :
path length to global optimum is also a criterium of difficulty

Properties of long k-path

- Unimodal problem :
  - Path length grows exp. with bitstring size
  - Hamming distance between two consecutive solutions is 1
  - Any other solution at distance 1 is off the path
- Very long path to global optimum
- "Short-cuts" require at least k bit-flips

Long 2-path $l = 7$



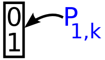
Results on long k-Path problems

Length

$|P_{l,k}| = (k+1)2^{(l-1)/k} - k + 1$ is the length of the k-path of dimension l.

Expected running time

- For a hillclimbing algorithm
  chooses the best solution in the neighborhood defined by Hamming distance 1 :
  $l \cdot |P_{l,k}|$
- For a $(1+1)$ EA
  flips each bit with a probability $p = 1/l$ at each iteration :
  $\mathcal{O}(l^{k+1}/k)$ [Rudolph, 1996]



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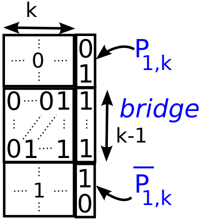

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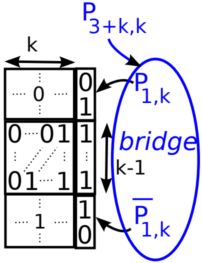

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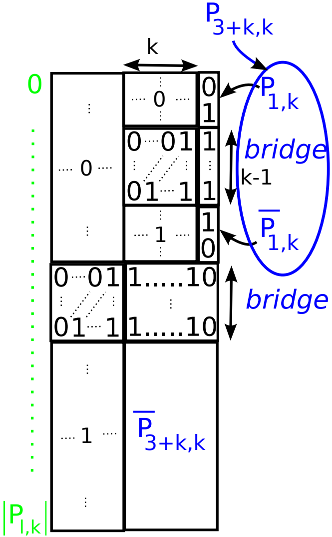
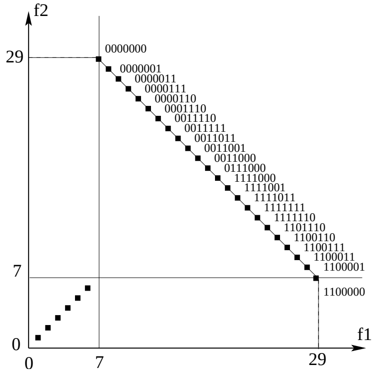


Bi-objective Long k-Path problems

Connected bi-objective problem : more difficult for multiobjective hillclimbing (PLS) than simple multiobjective EA

Long 2-Path with $l = 7$



Definition

Efficient set = long path $P_{l,k}$

Properties

- Intractable efficient set : $\Rightarrow$ eff. set approximation
- Running time of the PLS : exponential,
- Simple EA can take shortcuts

Multiobjective LS *vs.* EA

Pareto Local Search (PLS)

1. One solution is chosen at random from the archive.
2. *All solutions located at Hamming distance* 1 *are evaluated*
3. Check for insertion in the archive
4. The current solution is then marked as *visited*

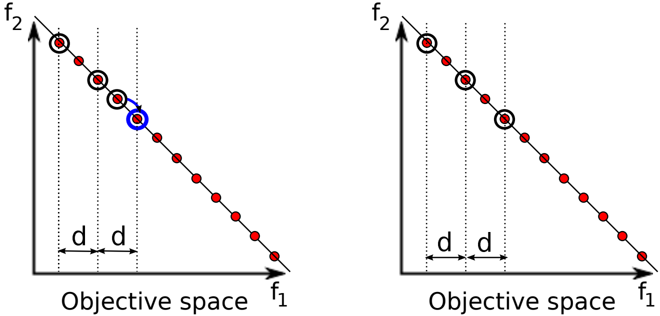
Simple Evolutionary Multiobj. Optimization (SEMO)

1. One solution is randomly chosen from the archive
2. *Each bit of this solution is independently flipped with a probability* $p = 1/l$
3. Check for insertion in the archive

All solutions are potentially reachable

- Starting from on the first solution of the long k-path
- Stop when 98% of the optimal hypervolume for archive of size 100 is reached

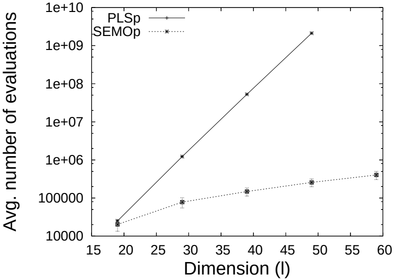
Limited size archive : "ideal" archive update



Archiving method

- Limited-size archive
- Reduce archive size such that $f_1(a_{i+1}) - f_1(a_i) \geq d$

Experimental results



Results

- Pareto LS : Number of evaluations grows exponentially with problem dimension
- Simple Multiobjective EA outperforms Pareto LS

First conclusions

- From the point of view of Pareto LS :
  - Efficient graph is linear,
  - Distance between efficient solutions is large
- From the point of view of SEMO :
  - Efficient graph : fully connected, high degree
  - Distance between efficient solutions is short

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First conclusions on difficulty

Connectedness
+ Structure of the efficient graph !

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First conclusions on difficulty

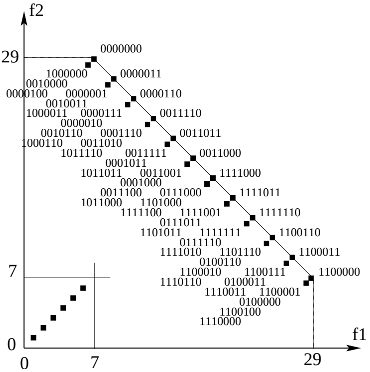
Connectedness
+ Structure of the efficient graph !

But it is still not enought...

Biobjective Multiple k-Path problems

Disconnected bi-objective problem : more difficult for simple multiobjective EA than multiobjective hillclimbing (PLS)

Multiple 2-Path with $l = 7$

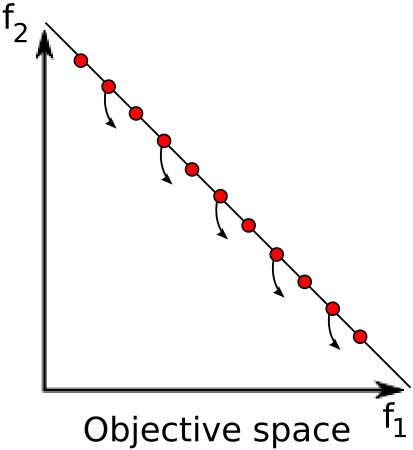


Definition

Modify the previous biobjective long k-Path :

- Disconnect the efficient graph
- Add some "dominate" paths between solutions

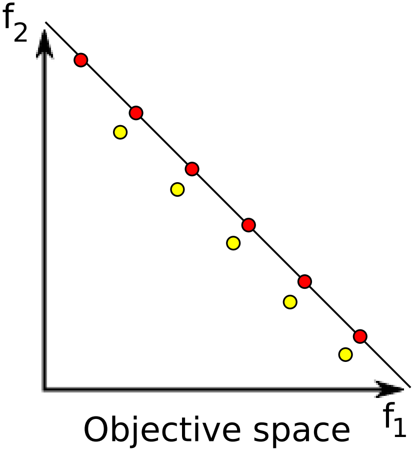
Biobjective Multiple k-Path problems



- Disconnect the efficient graph : move 1 over 2 solutions "under" the next solution in the path

example of bridges with $k = 3$

Biobjective Multiple k-Path problems



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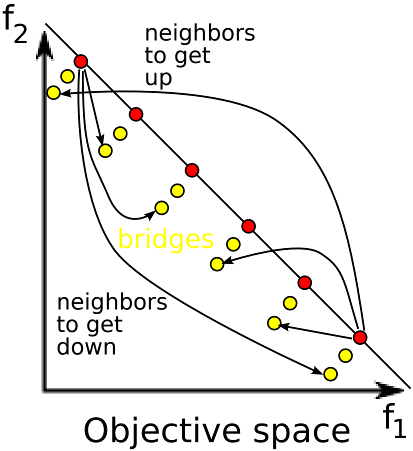
Biobjective Multiple k-Path problems



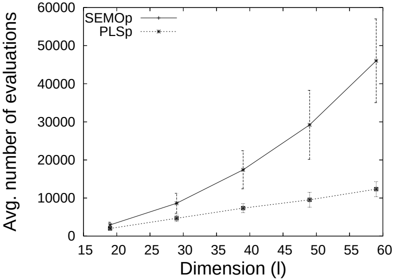
- Add recursively "dominate" paths between solutions :
  Add some bridges recursively

example of bridges with $k = 3$

$u0^k P(i_0)$	??????000.111.0
bridge	??????001.111.0
down	??????011.111.0
$u1^k P(i_1)$	??????111.111.0

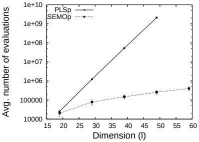
$u0^k P(i_0)$	??????000.111.0
bridge	??????100.111.0
up	??????110.111.0
$u1^k P(i_1)$	??????111.111.0

Experimental results

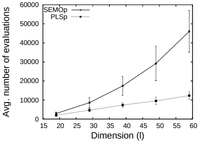


- Multiobjective LS : number of evaluations grows nearly linearly with problem dimension
- Pareto LS outperforms Simple Multiobjective EA

Summary on results



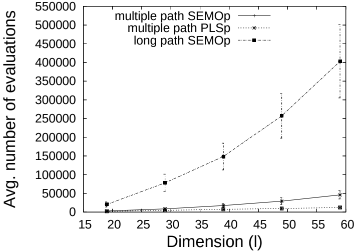
Long 2-path problem



Multiple 2-path problem

PLS <<< SEMO

PLS >> SEMO



long PLS <<< long SEMO <<< multiple SEMO << multiple PLS

Conclusions and future works

- Connectedness, and structure of efficient set is not the key concept to explain the difficulty

Reconsider relevant features of multiobjective problems

- Search space of multiobjective problems is the set of sets

$$\Sigma = \{\sigma \subset S \ : \ |\sigma| \leq n\}$$

- Dynamics of LS, and difficulty of problem instance could be explain in Σ (prob. to improve a set, local optimum sets, etc.)

Future works

- Running time analysis for the biobjective 'path' problems
- Study the fitness landscape (and dynamics of LS) where the search space is Σ

Bibliography

Ehrgott, M. (2005).
Multicriteria optimization.
Springer, second edition.

Ehrgott, M. and Klamroth, K. (1997).
Connectedness of efficient solutions in multiple criteria combinatorial optimization.
European Journal of Operational Research, 97(1) :159–166.

Gorski, J., Klamroth, K., and Ruzika, S. (2006).
Connectedness of efficient solutions in multiple objective combinatorial optimization.
Technical Report 102/2006, University of Kaiserslautern, Department of Mathematics.

Horn, J., Goldberg, D. E., and Deb, K. (1994).
Long path problems.
In *PPSN*, volume 866 of *LNCS*, pages 149–158. Springer.

Paquete, L., Chiarandini, M., and Stützle, T. (2004).
Pareto local optimum sets in the biobjective traveling salesman problem : An experimental study.
In *Metaheuristics for Multiobjective Optimisation*, volume 535 of *Lecture Notes in Economics and Mathematical Systems*, chapter 7, pages 177–199. Springer.

Rudolph, G. (1996).
How mutation and selection solve long path problems in polynomial expected time.
Evolutionary Computation, 4(2) :195–205.