Analysis on Selection for High Resolution Approximations in Many-objective Optimization

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Introduction

- In many-objective problems the number of Pareto optimal solutions increases exponentially with the number of objectives.

Many-objective problem solving

- All non-dominated solutions found are used for analysis and problem understanding
  - Often, all generated solutions are used
- Incremental observation scaling up number of objectives
- Observations across various problem formulations

Usual performance assessment in many-objective optimization

- Convergence and diversity
- Contents of the final population ?

Aim of the optimizer

- Convergence, diversity and resolution of the approximation

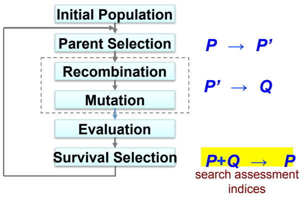
Objective

- Analyze the three elitist multi- and many-objective evolutionary algorithms generating a high-resolution approximation of the POS.

Algorithms

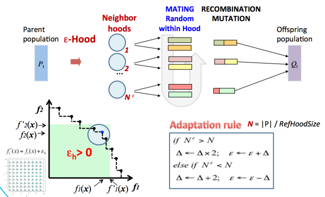
- **A**εSεH
- **I**BEA Iε+, κ=0.001
- **N**SGA-II

Same operators
- crossover
- mutation

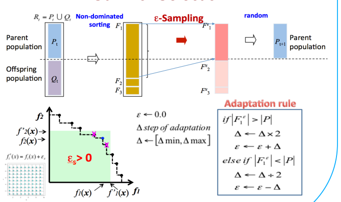


AεSεH

Parent Selection



Survival Selection



Analysis on MNK-landscapes

M	|POS|	Fronts	|P|/ |POS| (%)								
			50	100	200	500	1,000	2,000	4,000	5,600	11,200
3	152	258	32.9	65.8	132.6						
4	1,554	76	3.2	6.4	12.9	32.2	64.4				
5	6,265	29	0.8	1.6	3.2			31.9	63.8		
6	16,845	22	0.3	0.6	1.2					33.2	66.5

- Number of objectives vary, M = 3, 4, 5, 6
- Number of bits, N = 20
- Epistasis, K = 1

- Enumerate of solutions for analysis
- Population size: fraction of the POS
- Small landscapes ➡ easy to hit the POS

$$\boldsymbol{f}(\cdot) = (\, f_1(\cdot), \ldots, f_M(\cdot)\,) : \boldsymbol{B}^N \rightarrow \Re^M$$

M, number of objectives B={0,1}

N, number of bits

Fitness of string x in i-th objective [Kauffman, 1993]

$$f_i(x) = \frac{1}{N} \sum_{j=1}^{N} f_{i,j}\left(x_j, \underbrace{z_1^{(i,j)}, z_2^{(i,j)}, \ldots, z_{K_i}^{(i,j)}}\right)$$

$f_{i,j} : \boldsymbol{B}^{K_i+1} \rightarrow \Re$ K_i bits interacting with x_j

Fitness contribution of bit x_j

Method

Approximation of POS at generation t

$$\mathcal{A}(t) = \{\boldsymbol{x} : \boldsymbol{x} \in \mathcal{X}(t) = \mathcal{A}(t-1) \cup \mathcal{F}_1(t) \setminus \mathcal{A}(t-1) \cap \mathcal{F}_1(t) \wedge \nexists \boldsymbol{y} \in \mathcal{X}(t)\ \boldsymbol{y} \succeq \boldsymbol{x}\} \quad (1)$$

$$\mathcal{A}(0) = \mathcal{F}_1(0), \quad (2)$$

set of all non-dominated solutions found from generation 0 until generation t

Resolution of the Approximation

$$\alpha(t) = \frac{|\{\boldsymbol{x} : \boldsymbol{x} \in \mathcal{A}(t) \wedge \boldsymbol{x} \in POS\}|}{|POS|},$$

fraction of POS in the approximation to total number of POS

Population Gain

$$\beta(t) = \frac{|\{\boldsymbol{x} : \boldsymbol{x} \in \mathcal{A}(t) \wedge \boldsymbol{x} \in POS\}|}{|P|}.$$

fraction of POS in the approximation to population size

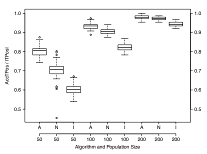
Generational Search Assessment Indices

I_t	Formula	Comment
τ_t	$\|\{\boldsymbol{x} : \boldsymbol{x} \in \mathcal{F}_1(t) \wedge \boldsymbol{x} \in POS\}\| \,/\, \|P\|$	PO solutions
τ_t^-	$\|\{\boldsymbol{x} : \boldsymbol{x} \in \mathcal{F}_1(t) \wedge \boldsymbol{x} \in \mathcal{F}_1(t-1) \wedge \boldsymbol{x} \in POS\}\| \,/\, \|P\|$	Old PO solutions
τ_t^+	$\|\{\boldsymbol{x} : \boldsymbol{x} \in \mathcal{F}_1(t) \wedge \boldsymbol{x} \notin \mathcal{F}_1(t-1) \wedge \boldsymbol{x} \in POS\}\| \,/\, \|P\|$	Possibly new PO solutions
τ_t^*	$\|\{\boldsymbol{x} : \boldsymbol{x} \in \mathcal{F}_1(t) \wedge \boldsymbol{x} \notin \cup_{k=1}^{t-1} \mathcal{F}_1(k) \wedge \boldsymbol{x} \in POS\}\| \,/\, \|P\|$	Absolutely new PO solutions
δ_t	$\|\{\boldsymbol{x} : \boldsymbol{x} \in \mathcal{F}_1(t-1) \wedge \boldsymbol{x} \notin \mathcal{F}_1(t) \wedge \boldsymbol{x} \in POS\}\| \,/\, \|P\|$	Dropped PO solutions
γ_t	$\|\{\boldsymbol{x} : \boldsymbol{x} \in \mathcal{F}_1(t) \wedge \boldsymbol{x} \notin POS\}\| \,/\, \|P\|$	Non-dominated, not PO sol.

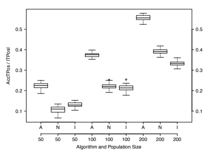
measures taken on non-dominated set F_1(t) with respect to F_1(t-1) and/or the POS, normalized by population size |P|

Results and Conclusions

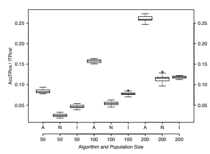
Resolution of the approximation α(T=100) Population size 50, 100, 200



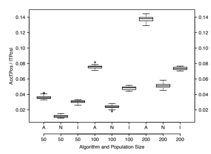
M=3 objectives



M=4 objectives

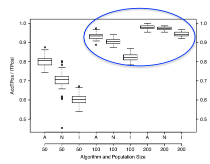


M=5 objectives

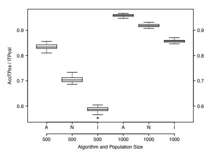


M=6 objectives

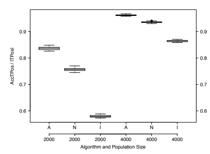
Resolution of the approximation α(T=100) Population size ~ 33% and 66% of |POS|



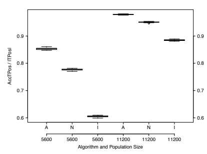
M=3 objectives



M=4 objectives

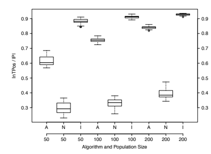


M=5 objectives

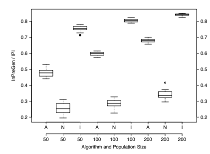


M=6 objectives

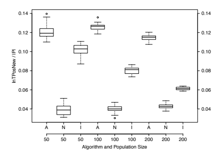
Average Search Assessment Indices (t=0,…,T) Population size 50, 100, 200, M=6



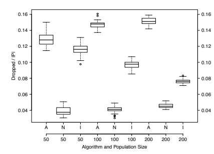
τ Pareto optimal



τ⁻ Old Pareto optimal

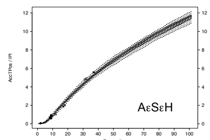


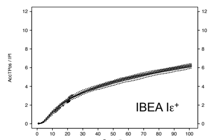
τ* Absolutely new Pareto optimal



δ Dropped Pareto optimal

Accumulated Population Gain Population size 200, M=6





- Indices are helpful to trace the dynamics and understand the behavior of the algorithms
- The study shows the importance of properly assess performance of many-objective optimizers
- Extend to larger landscapes